

AP Calculus AB Summer Packet (2019-20 School Year)

Every problem on this summer packet is **NON-CALCULATOR** unless otherwise stated and should be answered in **FRACTION FORM**, when applicable. You are responsible for knowing how to do everything on this packet on the first day of school! Do all work on a separate piece of paper. Keep your work organized and well-labeled!

Section 1: Linear Equations and Slope

- Find the slope of $2x + 4y = 5$.
- A line passes through $(2,3)$ and has a slope of 4. Find the equation of the line in both point-slope and slope-intercept form.
- A line passes through $(4, -2)$ and $(1,2)$. Find the equation of the line in both point-slope and slope-intercept form.
- Find the equation of the line parallel to $x - 2y = 12$ and passing through $(2, -5)$. Find the equation of the line in both point-slope and slope-intercept form.
- Find the equation of the line perpendicular to $3x + 4y = -8$ and passing through $(0, -2)$. Find the equation of the line in both point-slope and slope-intercept form.
- Find the average rate of change of $f(x) = x^2 - 4x + 3$ between $x = 0$ and $x = 1$.

Section 2: Asymptotes and Discontinuities

State the vertical and horizontal asymptotes for each of the following graphs.

- $y = \frac{2}{x-3}$
- $y = \frac{x^2}{x^2-4}$
- $y = \ln(x - 3)$
- $y = e^{x-3} + 1$

Section 3: Domain and Range

State the domain and range of each of the following using either set notation or interval notation.

- $y = e^{-2x} + 1$
- $y = \ln(x + 2)$
- $y = \tan^{-1}(x)$
- $y = \cos(x)$
- $y = \sin(\arctan(x))$
- $y = \ln(\cos(x))$
- $y = \cos(\ln(x))$
- $y = \frac{1}{x}$

Section 4: Trigonometry

Find the value of each:

- | | | | |
|--|---------------------------------------|---|--|
| 19. $\sin\left(\frac{2\pi}{3}\right)$ | 20. $\cos\left(\frac{5\pi}{3}\right)$ | 21. $\sin(\pi)$ | 22. $\cos\left(-\frac{7\pi}{6}\right)$ |
| 23. $\sec^2\left(\frac{\pi}{6}\right)$ | 24. $\tan\left(\frac{\pi}{4}\right)$ | 25. $\tan^2\left(\frac{5\pi}{6}\right)$ | 26. $\arccos(-1)$ |
| 27. $\tan\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)$ | | | |

28. Find $\csc(\theta)$ given $\cos(\theta) = \frac{12}{13}$ and $\tan(\theta) < 0$.
29. Find the smallest positive value x for which $f(x) = \sin\left(\frac{x}{3}\right) - 1$ is a maximum.
30. Solve $4\sin^2(x) - 1 = 0$
31. Solve $\tan^2(x) + \tan(x) = 0$
32. Solve $\sin(2x) = 0.5$
33. Solve $\sec(x) - 2 = 0$
34. Solve $2\sin^2(x) + \sin(x) = 1$
35. Given $\sin(\theta) = \frac{4\sqrt{5}}{9}$ and $\sec(\theta) = 6$, find $\tan(\theta)$.
36. Sketch $\triangle ABC$ and solve for all angles given $a = 13$, $b = 18$, $c = 26$. (Calculator Allowed)

Section 5: Inverse Functions and Relations

37. Find $f^{-1}(x)$ given $f(x) = 2\sqrt{x}$.
38. Find $f^{-1}(x)$ given $f(x) = 2e^{-x}$.
39. If point $(5, -2)$ is on the graph of f , what point must be on the graph of f^{-1} ?

Section 6: Polynomials (Factoring, Completing the Square, Compositions, and Synthetic Division)

40. Given $f(x) = 2x^2 - 3x$, find $\frac{f(2+h)-f(2)}{h}$.
41. Factor $x^2 + 12x - 45$
42. Factor $4x^2 - 16y^4$
43. Factor $2x^2 - x - 3$
44. Divide using synthetic division: $\frac{x^3 - 27x + 54}{x - 3}$
45. Convert to vertex form by completing the square: $y = x^2 + 8x + 1$
46. Convert to vertex form by completing the square: $y = 2x^2 - 12x + 7$

Section 7: Logs and Exponentials

Solve and/or simplify:

47. $\log_3(x - 4) = 0$
48. $\ln(x^4) = 10$
49. $e^{2x} - e^x = 6$
50. $\ln(6x - 6) = 2$

51. $\log_4(2) + \log_4(8) = \log_4(x)$
52. Identify the x and y intercepts of $f(x) = 4 \ln(x - 2)$
53. Expand the following log:
- $$\ln\left(\frac{x^2}{\sqrt{x^3+1}}\right)$$
54. Condense the following into a single log:
- $$2(\log_5 x + \log_5 7) - \frac{1}{3}\log_5 y$$
55. Find the domain of the function $f(x) = \ln\left(\frac{x-3}{x+2}\right)$

Section 8: Limits

56. $\lim_{x \rightarrow 4} \frac{x-4}{x^2-x-12}$
57. $\lim_{x \rightarrow 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$
58. $\lim_{x \rightarrow 2} \frac{x-2}{x^2+4}$

Section 9: Graph Sketching

Sketch a graph of each of the following functions. Make sure you show key characteristics of each graph. Include key point(s) if applicable.

59. $y = e^x$ 60. $y = \ln(x)$ 61. $y = \sin(x)$ 62. $y = \cos(x)$
63. $y = \tan(x)$ 64. $y = \tan^{-1}(x)$ 65. $y = \sqrt{x}$ 66. $y = x^2$
67. $y = x^3$ 68. $y = 2^x$ 69. $(x - 3)^2 + (y - 2)^2 = 25$

Section 10: Graphing Calculator Practice (Calculator Allowed...obviously)

Given $f(x) = 1.332x^3 - 4.225x^2 + 1.130x + 1.700$ and $g(x) = e^{\sin(x)}$, find each of the following values. Round to the nearest thousandth.

70. The only relative maximum value of f .
71. The only relative minimum value of f .
72. All x-intercepts of g on the interval $-1 \leq x \leq 6$.
73. The x-value of all intersections of f and g on the interval $0 < x < 5$.
74. At what value of x is the absolute difference between f and g at a maximum on the interval $-0.5 < x < 2.5$?
75. The instantaneous slope of g at the point where $x = 1$. (BONUS IF YOU CAN GET THIS ONE RIGHT!)