Miscellaneous
Multiple-Choice
Practice Questions

These questions provide further practice for Parts A and B of Section I of the examination. Answers begin on page 470.

Part A. Directions: Answer these questions without using your calculator.

1. Which of the following functions is continuous at $x = 0$?

   (A) $f(x) = \begin{cases} \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$  
   (B) $f(x) = [x]$ (greatest-integer function)

   (C) $f(x) = \begin{cases} \frac{x}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$  
   (D) $f(x) = \begin{cases} \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

   (E) $f(x) = \frac{x+1}{x}$

2. Which of the following statements about the graph of $y = \frac{x^2 + 1}{x^2 - 1}$ is not true?

   (A) The graph is symmetric to the y-axis.
   (B) The graph has two vertical asymptotes.
   (C) There is no y-intercept.
   (D) The graph has one horizontal asymptote.
   (E) There is no x-intercept.

3. $\lim_{x \to 0} ([x] - |x|) =$

   (A) $-1$  (B) $0$  (C) $1$  (D) $2$  (E) none of these

4. The $x$-coordinate of the point on the curve $y = x^2 - 2x + 3$ at which the tangent is perpendicular to the line $x + 3y + 3 = 0$ is

   (A) $-\frac{5}{2}$  (B) $-\frac{1}{2}$  (C) $\frac{7}{6}$  (D) $\frac{5}{2}$  (E) none of these

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5. \( \lim_{x \to 1} \frac{3}{x - 3} \) is
   (A) -3   (B) -1   (C) 1   (D) 3   (E) nonexistent

6. \( \ln (3e^{x^2}) - \ln (3e^x) = \)
   (A) 2   (B) \( e^x \)   (C) 2 \( \ln 3 \)   (D) \( \ln 3 + 2 \)   (E) \( 2 \ln 3 + 2 \)

7. \( \int_0^6 |x - 4| \, dx = \)
   (A) 6   (B) 8   (C) 10   (D) 11   (E) 12

8. \( \lim_{x \to \infty} \frac{3 + x - 2x^2}{4x^2 + 9} \) is
   (A) \( -\frac{1}{2} \)   (B) \( \frac{1}{2} \)   (C) 1   (D) 3   (E) nonexistent

9. The maximum value of the function \( f(x) = x^4 - 4x^3 + 6 \) on the closed interval \([1, 4]\) is
   (A) 1   (B) 0   (C) 3   (D) 6   (E) none of these

10. Let \( f(x) = \frac{\sqrt{x + 4} - 3}{x - 5} \) if \( x \neq 5 \), and let \( f \) be continuous at \( x = 5 \). Then \( c = \)
    (A) \( -\frac{1}{6} \)   (B) 0   (C) \( \frac{1}{6} \)   (D) 1   (E) 6

11. \( \int_0^\pi \cos^2 x \sin x \, dx = \)
    (A) -1   (B) \( -\frac{1}{3} \)   (C) 0   (D) \( \frac{1}{3} \)   (E) 1

12. If \( \sin x = \ln y \) and \( 0 < x < \pi \), then, in terms of \( x \), \( \frac{dy}{dx} \) equals
    (A) \( e^{\sin x} \cos x \)   (B) \( e^{-\sin x} \cos x \)   (C) \( \frac{e^{\sin x}}{\cos x} \)
    (D) \( e^{\cos x} \)   (E) \( e^{\sin x} \)

13. If \( f(x) = x \cos x \), then \( f' \left( \frac{\pi}{2} \right) \) equals
    (A) \( \frac{\pi}{2} \)   (B) 0   (C) -1   (D) \( -\frac{\pi}{2} \)   (E) 1
14. The equation of the tangent to the curve \( y = e^x \ln x \), where \( x = 1 \), is
   \[ \begin{align*}
   \text{(A) } & y = ex \quad \text{(B) } y = e^x + 1 \quad \text{(C) } y = e(x - 1) \\
   \text{(D) } & y = ex + 1 \quad \text{(E) } y = x - 1
   \end{align*} \]

15. If the displacement from the origin of a particle moving along the \( x \)-axis is given by \( s = 3 + (t - 2)^4 \), then the number of times the particle reverses direction is
   \[ \begin{align*}
   \text{(A) } & 0 \quad \text{(B) } 1 \quad \text{(C) } 2 \quad \text{(D) } 3 \quad \text{(E) none of these}
   \end{align*} \]

16. \( \int_{-1}^{0} e^{-x} \, dx \) equals
   \[ \begin{align*}
   \text{(A) } & 1 - e \quad \text{(B) } \frac{1 - e}{e} \quad \text{(C) } e - 1 \quad \text{(D) } 1 - \frac{1}{e} \quad \text{(E) } e + 1
   \end{align*} \]

17. If \( f(x) = \begin{cases} x^2 & \text{for } x \leq 2 \\ 4x - x^2 & \text{for } x > 2 \end{cases} \), then \( \int_{-1}^{4} f(x) \, dx \) equals
   \[ \begin{align*}
   \text{(A) } & 7 \quad \text{(B) } \frac{23}{3} \quad \text{(C) } \frac{25}{3} \quad \text{(D) } 9 \quad \text{(E) } \frac{65}{3}
   \end{align*} \]

18. If the position of a particle on a line at time \( t \) is given by \( s = t^3 + 3t \), then the speed of the particle is decreasing when
   \[ \begin{align*}
   \text{(A) } & -1 < t < 1 \quad \text{(B) } -1 < t < 0 \quad \text{(C) } t < 0 \\
   \text{(D) } & t > 0 \quad \text{(E) } |t| > 1
   \end{align*} \]

19. A rectangle with one side on the \( x \)-axis is inscribed in the triangle formed by the lines \( y = x \), \( y = 0 \), and \( 2x + y = 12 \). The area of the largest such rectangle is
   \[ \begin{align*}
   \text{(A) } & 6 \quad \text{(B) } 3 \quad \text{(C) } \frac{5}{2} \quad \text{(D) } 5 \quad \text{(E) } 7
   \end{align*} \]

20. The abscissa of the first-quadrant point that is on the curve of \( x^2 - y^2 = 1 \) and closest to the point \((3, 0)\) is
   \[ \begin{align*}
   \text{(A) } & 1 \quad \text{(B) } \frac{3}{2} \quad \text{(C) } 2 \quad \text{(D) } 3 \quad \text{(E) none of these}
   \end{align*} \]

21. If \( y = \ln(4x + 1) \), then \( \frac{d^2 y}{dx^2} \) is
   \[ \begin{align*}
   \text{(A) } & \frac{1}{4} \quad \text{(B) } \frac{-1}{(4x+1)^2} \quad \text{(C) } \frac{-4}{(4x+1)^2} \\
   \text{(D) } & \frac{-16}{(4x+1)^2} \quad \text{(E) } \frac{-1}{16(4x+1)^2}
   \end{align*} \]

22. The region bounded by the parabolas \( y = x^2 \) and \( y = 6x - x^2 \) is rotated about the \( x \)-axis so that a vertical line segment cut off by the curves generates a ring. The value of \( x \) for which the ring of largest area is obtained is
   \[ \begin{align*}
   \text{(A) } & 4 \quad \text{(B) } 3 \quad \text{(C) } \frac{5}{2} \quad \text{(D) } 2 \quad \text{(E) } \frac{3}{2}
   \end{align*} \]
23. \( \int \frac{dx}{\ln x} \) equals

(A) \( \ln(\ln x) + C \)  \hspace{1cm} (B) \( -\frac{1}{\ln^2 x} + C \)  \hspace{1cm} (C) \( \frac{(\ln x)^2}{2} + C \)

(D) \( \ln x + C \)  \hspace{1cm} (E) none of these

24. The volume obtained by rotating the region bounded by \( x = y^2 \) and \( x = 2 - y^2 \) about the y-axis is equal to

(A) \( \frac{16\pi}{3} \)  \hspace{1cm} (B) \( \frac{32\pi}{3} \)  \hspace{1cm} (C) \( \frac{32\pi}{15} \)  \hspace{1cm} (D) \( \frac{64\pi}{15} \)  \hspace{1cm} (E) \( \frac{8\pi}{3} \)

25. The general solution of the differential equation \( \frac{dy}{dx} = \frac{1 - 2x}{y} \) is a family of

(A) straight lines \hspace{1cm} (B) circles \hspace{1cm} (C) hyperbolas

(D) parabolas \hspace{1cm} (E) ellipses

26. Estimate \( \int_{0}^{4} \sqrt{25 - x^2} \, dx \) using the Left Rectangular Rule and two subintervals.

(A) \( 3 + \sqrt{21} \)  \hspace{1cm} (B) \( 5 + \sqrt{21} \)  \hspace{1cm} (C) \( 6 + 2\sqrt{21} \)

(D) \( 8 + 2\sqrt{21} \)  \hspace{1cm} (E) \( 10 + 2\sqrt{21} \)

27. \( \int_{0}^{7} \sin \pi x \, dx = \)

(A) \(-2\)  \hspace{1cm} (B) \(-\frac{2}{\pi}\)  \hspace{1cm} (C) \(0\)  \hspace{1cm} (D) \(\frac{1}{\pi}\)  \hspace{1cm} (E) \(\frac{2}{\pi}\)

28. \( \lim_{x \to 0} \frac{\tan 3x}{2x} = \)

(A) \(0\)  \hspace{1cm} (B) \(\frac{1}{2}\)  \hspace{1cm} (C) \(\frac{2}{3}\)  \hspace{1cm} (D) \(\frac{3}{2}\)  \hspace{1cm} (E) \(\infty\)

29. \( \lim_{h \to 0} \frac{\tan(\pi / 4 + h) - 1}{h} = \)

(A) \(0\)  \hspace{1cm} (B) \(\frac{1}{2}\)  \hspace{1cm} (C) \(1\)  \hspace{1cm} (D) \(2\)  \hspace{1cm} (E) \(\infty\)

30. The number of values of \( k \) for which \( f(x) = e^x \) and \( g(x) = k \sin x \) have a common point of tangency is

(A) \(0\)  \hspace{1cm} (B) \(1\)  \hspace{1cm} (C) \(2\)  \hspace{1cm} (D) large but finite  \hspace{1cm} (E) infinite
31. The curve $2x^2y + y^2 = 2x + 13$ passes through $(3, 1)$. Use the local linearization of the curve to find the approximate value of $y$ at $x = 2.8$.

(A) 0.5 (B) 0.9 (C) 0.95 (D) 1.1 (E) 1.4

32. $\int \cos^3 x \, dx =$

(A) $\frac{\cos^4 x}{4} + C$ (B) $\frac{\sin^4 x}{4} + C$ (C) $\sin x - \frac{\sin^3 x}{3} + C$

(D) $\sin x + \frac{\sin^3 x}{3} + C$ (E) $\cos x + \frac{\cos^3 x}{3} + C$

33. The region bounded by $y = \tan x$, $y = 0$, and $x = \frac{\pi}{4}$ is rotated about the $x$-axis. The volume generated equals

(A) $\pi = \frac{\pi^2}{4}$ (B) $\pi(\sqrt{2} - 1)$ (C) $\frac{3\pi}{4}$

(D) $\pi \left(1 + \frac{\pi}{4}\right)$ (E) none of these

34. $\lim_{h \to 0} \frac{a^h - 1}{h}$, for the constant $a > 0$, equals

(A) 1 (B) $a$ (C) $\ln a$ (D) $\log_{10} a$ (E) $a \ln a$

35. Solutions of the differential equation whose slope field is shown here are most likely to be

(A) quadratic (B) cubic (C) sinusoidal (D) exponential (E) logarithmic

36. $\lim_{h \to 0} \frac{1}{h} \int_{\frac{\pi}{4}}^{\frac{\pi}{2} + h} \frac{\sin x}{x} \, dx =$

(A) 0 (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{2\sqrt{2}}{\pi}$ (E) $\frac{2\sqrt{2}(\pi - 4)}{\pi^2}$
37. The graph of \( g \), shown below, consists of the arcs of two quarter-circles and two straight-line segments. The value of \( \int_{0}^{12} g(x) \, dx \) is

\[ \begin{align*}
(A) \; \pi + 2 & \quad (B) \; \frac{7\pi}{2} + \frac{9}{2} \\
(D) \; 7\pi + \frac{9}{2} & \quad (E) \; \frac{25\pi}{4} + \frac{21}{2}
\end{align*} \]

38. Which of these could be a particular solution of the differential equation whose slope field is shown here?

\[ \begin{align*}
(A) \; y = \frac{1}{x} & \quad (B) \; y = \ln x \\
(C) \; y = e^x & \quad (D) \; y = e^{-x} \\
(E) \; y = e^x
\end{align*} \]
39. The slope field shown here is for the differential equation

\[ (A) \quad y' = \frac{1}{x} \quad (B) \quad y' = \ln x \quad (C) \quad y' = e^t \quad (D) \quad y' = y \quad (E) \quad y' = -y^2 \]

40. If we substitute \( x = \tan \theta \), which of the following is equivalent to \( \int_0^1 \sqrt{1 + x^2} \, dx \)?

\[ (A) \quad \int_0^1 \sec \theta \, d\theta \quad (B) \quad \int_0^1 \sec^3 \theta \, d\theta \quad (C) \quad \int_0^{\pi/4} \sec \theta \, d\theta \]
\[ (D) \quad \int_0^{\pi/4} \sec^3 \theta \, d\theta \quad (E) \quad \int_0^{\tan 1} \sec^3 \theta \, d\theta \]

41. If \( x = 2 \sin u \) and \( y = \cos 2u \), then a single equation in \( x \) and \( y \) is

\[ (A) \quad x^2 + y^2 = 1 \quad (B) \quad x^2 + 4y^2 = 4 \quad (C) \quad x^2 + 2y = 2 \]
\[ (D) \quad x^2 + y^2 = 4 \quad (E) \quad x^2 - 2y = 2 \]

42. The area bounded by the lemniscate with polar equation \( r^2 = 2 \cos 2\theta \) is equal to

\[ (A) \quad 4 \quad (B) \quad 1 \quad (C) \quad \frac{1}{4} \quad (D) \quad 2 \quad (E) \quad \text{none of these} \]

43. \( \int_{-\infty}^{\infty} \frac{dx}{x^4 + 1} = \)

\[ (A) \quad 0 \quad (B) \quad \frac{\pi}{4} \quad (C) \quad \pi \quad (D) \quad 2\pi \quad (E) \quad \text{none of these} \]

44. The first four terms of the Maclaurin series (the Taylor series about \( x = 0 \)) for

\[ f(x) = \frac{1}{1 - 2x} \] are

\[ (A) \quad 1 + 2x + 4x^2 + 8x^3 \quad (B) \quad 1 - 2x + 4x^2 - 8x^3 \]
\[ (C) \quad -1 - 2x - 4x^2 - 8x^3 \quad (D) \quad 1 - x + x^2 - x^3 \]
\[ (E) \quad 1 + x + x^2 + x^3 \]

45. \( \int x^2 e^{-x} \, dx = \)

\[ (A) \quad -\frac{1}{3} x^3 e^{-x} + C \quad (B) \quad \frac{1}{3} x^3 e^{-x} + C \quad (C) \quad -x^2 e^{-x} + 2xe^{-x} + C \]
\[ (D) \quad -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + C \quad (E) \quad -x^2 e^{-x} + 2xe^{-x} - 2e^{-x} + C \]
46. \( \int_0^{\pi/2} \sin^2 x \, dx \) is equal to
   
   (A) \( \frac{1}{3} \)  \hspace{1cm} (B) \( \frac{\pi}{4} - \frac{1}{4} \)  \hspace{1cm} (C) \( \frac{\pi}{3} \)  \hspace{1cm} (D) \( \frac{\pi}{2} - \frac{1}{3} \)  \hspace{1cm} (E) \( \frac{\pi}{4} \)

47. A curve is given parametrically by the equations \( x = t, \ y = 1 - \cos t \). The area bounded by the curve and the \( x \)-axis on the interval \( 0 \leq t \leq 2\pi \) is equal to
   
   (A) \( 2(\pi + 1) \)  \hspace{1cm} (B) \( \pi \)  \hspace{1cm} (C) \( 4\pi \)  \hspace{1cm} (D) \( \pi + 1 \)  \hspace{1cm} (E) \( 2\pi \)

48. If \( x = a \cot \theta \) and \( y = a \sin^2 \theta \), then \( \frac{dy}{dx} \), when \( \frac{\pi}{4} \), is equal to
   
   (A) \( \frac{1}{2} \)  \hspace{1cm} (B) \( -1 \)  \hspace{1cm} (C) \( 2 \)  \hspace{1cm} (D) \( -\frac{1}{2} \)  \hspace{1cm} (E) \( -\frac{1}{4} \)

49. Which of the following improper integrals diverges?
   
   (A) \( \int_0^\infty e^{-x^2} \, dx \)  \hspace{1cm} (B) \( \int_{-\infty}^0 e^x \, dx \)  \hspace{1cm} (C) \( \int_0^1 \frac{dx}{x} \)  \hspace{1cm} (D) \( \int_0^\infty e^{-x} \, dx \)  \hspace{1cm} (E) \( \int_0^1 \frac{dx}{\sqrt{x}} \)

50. \( \int_2^4 \frac{dt}{\sqrt{16 - t^2}} \) equals
   
   (A) \( \frac{\pi}{12} \)  \hspace{1cm} (B) \( \frac{\pi}{6} \)  \hspace{1cm} (C) \( \frac{\pi}{4} \)  \hspace{1cm} (D) \( \frac{\pi}{3} \)  \hspace{1cm} (E) \( \frac{2\pi}{3} \)

51. \( \lim_{x \to 0} \left( \frac{1}{x} \right)^x \)
   
   (A) \( -\infty \)  \hspace{1cm} (B) \( 0 \)  \hspace{1cm} (C) \( 1 \)  \hspace{1cm} (D) \( \infty \)  \hspace{1cm} (E) non-existent

52. A particle moves along the parabola \( x = 3y - y^2 \) so that \( \frac{dy}{dt} = 3 \) at all time \( t \).
   
   The speed of the particle when it is at position \( (2, 1) \) is equal to
   
   (A) \( 0 \)  \hspace{1cm} (B) \( 3 \)  \hspace{1cm} (C) \( \sqrt{13} \)  \hspace{1cm} (D) \( 3\sqrt{2} \)  \hspace{1cm} (E) none of these

53. \( \lim_{x \to 0^+} \frac{\cot x}{\ln x} = \)
   
   (A) \( -\infty \)  \hspace{1cm} (B) \( -1 \)  \hspace{1cm} (C) \( 0 \)  \hspace{1cm} (D) \( 1 \)  \hspace{1cm} (E) \( \infty \)

54. When rewritten as partial fractions, \( \frac{3x + 2}{x^2 - x - 12} \) includes which of the following?

   I. \( \frac{1}{x + 3} \)  \hspace{1cm} II. \( \frac{1}{x - 4} \)  \hspace{1cm} III. \( \frac{2}{x - 4} \)

   (A) none  \hspace{1cm} (B) I only  \hspace{1cm} (C) II only  \hspace{1cm} (D) III only  \hspace{1cm} (E) I and III
55. Using two terms of an appropriate Maclaurin series, estimate \( \int_0^{11} \frac{1 - \cos x}{x} \, dx \).

(A) \( \frac{1}{96} \)  
(B) \( \frac{23}{96} \)  
(C) \( \frac{1}{4} \)  
(D) \( \frac{25}{96} \)  
(E) undefined; the integral is improper

56. The slope of the spiral \( \frac{\pi}{4} \) is

(A) \( -\sqrt{2} \)  
(B) \( -1 \)  
(C) \( 1 \)  
(D) \( \frac{4 + \pi}{4 - \pi} \)  
(E) undefined

Part B. Directions: Some of these questions require the use of a graphing calculator.

57. The graph of function \( h \) is shown here. Which of these statements is (are) true?

I. The first derivative is never negative.
II. The second derivative is constant.
III. The first and second derivatives equal 0 at the same point.

(A) I only  
(B) III only  
(C) I and II  
(D) I and III  
(E) all three

58. Graphs of functions \( f(x) \), \( g(x) \), and \( h(x) \) are shown below. Consider the following statements:

I. \( g(x) = f'(x) \)
II. \( f(x) = g'(x) \)
III. \( h(x) = g''(x) \)

Which of these statements is (are) true?

(A) I only  
(B) II only  
(C) II and III only  
(D) all three  
(E) none of these
59. If \( y = \int_{3}^{x} \frac{1}{\sqrt{3+2t}} \, dt \), then \( \frac{d^2y}{dx^2} = \)

- (A) \( \frac{1}{(3+2x)^\frac{3}{2}} \)
- (B) \( \frac{3}{(3+2x)^\frac{3}{2}} \)
- (C) \( \frac{\sqrt{3+2x}}{2} \)
- (D) \( \frac{(3+2x)^\frac{3}{2}}{3} \)
- (E) 0

60. If \( \int_{3}^{4} f(x) \, dx = 6 \), then \( \int_{3}^{4} f(x+1) \, dx = \)

- (A) \(-6\)
- (B) \(-5\)
- (C) \(5\)
- (D) \(6\)
- (E) \(7\)

61. At what point in the interval [1, 1.5] is the rate of change of \( f(x) = \sin x \) equal to its average rate of change on the interval?

- (A) 0.995
- (B) 1.058
- (C) 1.239
- (D) 1.253
- (E) 1.399

62. Suppose \( f''(x) = x^2(x-1) \). Then \( f''(x) = x(3x-2) \). Over which interval(s) is the graph of \( f \) both increasing and concave up?

- I. \( x < 0 \)
- II. \( 0 < x < \frac{2}{3} \)
- III. \( \frac{2}{3} < x < 1 \)
- IV. \( x > 1 \)

- (A) I only
- (B) II only
- (C) II and IV
- (D) I and IV
- (E) IV only

63. Which of the following statements is true about the graph of \( f(x) \) in Question 62?

- (A) The graph has no relative extrema.
- (B) The graph has one relative extremum and one inflection point.
- (C) The graph has two relative extrema and one inflection point.
- (D) The graph has two relative extrema and two inflection points.
- (E) None of the preceding statements is true.

64. The \( n \)th derivative of \( \ln (x + 1) \) at \( x = 2 \) equals

- (A) \( \frac{(-1)^{n-1}}{3^n} \)
- (B) \( \frac{(-1)^n \cdot n!}{3^{n+1}} \)
- (C) \( \frac{(-1)^{n-1} \cdot (n-1)!}{3^n} \)
- (D) \( \frac{(-1)^{n+1} \cdot n!}{3^{n+1}} \)
- (E) \( \frac{(-1)^{n+1}}{3^{n+1}} \)

65. If \( f(x) \) is continuous at the point where \( x = a \), which of the following statements may be false?

- (A) \( \lim_{x \to a} f(x) \) exists.
- (B) \( \lim_{x \to a} f(x) = f(a) \).
- (C) \( f'(a) \) exists.
- (D) \( f(a) \) is defined.
- (E) \( \lim_{x \to a} f(x) = \lim_{x \to a^+} f(x) \).
66. Suppose \( \int_0^3 f(x + k) \, dx = 4 \), where \( k \) is a constant. Then \( \int_0^{3k} f(x) \, dx \) equals

(A) 3  \hspace{1cm} (B) 4 - k  \hspace{1cm} (C) 4  \hspace{1cm} (D) 4 + k  \hspace{1cm} (E) none of these

67. The volume, in cubic feet, of an “inner tube” with inner diameter 4 ft and outer diameter 8 ft is

(A) 4\( \pi^2 \)  \hspace{1cm} (B) 12\( \pi^2 \)  \hspace{1cm} (C) 8\( \pi^2 \)  \hspace{1cm} (D) 24\( \pi^2 \)  \hspace{1cm} (E) 6\( \pi^2 \)

68. If \( f(u) = \tan^{-1} u^2 \) and \( g(u) = e^u \), then the derivative of \( f(g(u)) \) is

(A) \( \frac{2ue^{u^2}}{1 + u^4} \)  \hspace{1cm} (B) \( \frac{2ue^{u^2}}{1 + u^4} \)  \hspace{1cm} (C) \( \frac{2e^u}{1 + 4u^2} \)

(D) \( \frac{2e^{2u}}{1 + e^{4u}} \)  \hspace{1cm} (E) \( \frac{2e^{2u}}{\sqrt{1 - e^{4u}}} \)

69. If \( \sin(xy) = y \), then \( \frac{dy}{dx} \) equals

(A) sec (xy)  \hspace{1cm} (B) \( y \cos(xy) - 1 \)  \hspace{1cm} (C) \( \frac{1 - y \cos(xy)}{x \cos(xy)} \)

(D) \( \frac{y \cos(xy)}{1 - x \cos(xy)} \)  \hspace{1cm} (E) \( \cos(xy) \)

70. Let \( x > 0 \). Suppose \( \frac{df}{dx} = g(x) \) and \( \frac{d^2 f}{dx^2} = f(\sqrt{x}) \); then \( \frac{d^2 f}{dx^2} f(x^2) = \)

(A) \( f(x^2) \)  \hspace{1cm} (B) \( f(x^2) \)  \hspace{1cm} (C) 2xg(x^2)

(D) \( \frac{1}{2x} f(x) \)  \hspace{1cm} (E) 2g(x^2) + 4x^2f(x)

71. The region bounded by \( y = e^x \), \( y = 1 \), and \( x = 2 \) is rotated about the \( x \)-axis. The volume of the solid generated is given by the integral

(A) \( \pi \int_0^2 e^{2x} \, dx \)  \hspace{1cm} (B) \( 2\pi \int_1^2 (2 - \ln y)(y - 1) \, dy \)  \hspace{1cm} (C) \( \pi \int_0^2 (e^{2x} - 1) \, dx \)

(D) \( 2\pi \int_0^e y(2 - \ln y) \)  \hspace{1cm} (E) \( \pi \int_0^2 (e^x - 1)^2 \, dx \)

72. Suppose the function \( f \) is continuous on \( 1 \leq x \leq 2 \), that \( f'(x) \) exists on \( 1 < x < 2 \), that \( f(1) = 3 \), and that \( f(2) = 0 \). Which of the following statements is not necessarily true?

(A) The Mean-Value Theorem applies to \( f \) on \( 1 \leq x \leq 2 \).

(B) \( \int_0^2 f(x) \, dx \) exists.

(C) There exists a number \( c \) in the closed interval \([1, 2]\) such that \( f'(c) = 0 \).

(D) If \( k \) is any number between 0 and 3, there is a number \( c \) between 1 and 2 such that \( f(c) = k \).

(E) If \( c \) is any number such that \( 1 < c < 2 \), then \( \lim_{x \to c} f(x) \) exists.
73. The region S in the figure is bounded by \( y = \sec x \), the \( y \)-axis, and \( y = 4 \). What is the volume of the solid formed when \( S \) is rotated about the \( y \)-axis?

(A) 0.791 (B) 2.279 (C) 5.692 (D) 11.385 (E) 17.217

74. If 40 g of a radioactive substance decomposes to 20 g in 2 yr, then, to the nearest gram, the amount left after 3 yr is

(A) 10 (B) 12 (C) 14 (D) 16 (E) 17

75. An object in motion along a line has acceleration \( a(t) = \pi t + \frac{2}{1+t^2} \) and is at rest when \( t = 1 \). Its average velocity from \( t = 0 \) to \( t = 2 \) is

(A) 0.362 (B) 0.274 (C) 3.504 (D) 7.008 (E) 8.497

76. Find the area bounded by \( y = \tan x \) and \( x + y = 2 \), and above the \( x \)-axis on the interval \([0, 2]\).

(A) 0.919 (B) 0.923 (C) 1.013 (D) 1.077 (E) 1.494

77. An ellipse has major axis 20 and minor axis 10. Rounded off to the nearest integer, the maximum area of an inscribed rectangle is

(A) 50 (B) 79 (C) 80 (D) 82 (E) 100

78. The average value of \( y = x \ln x \) on the interval \( 1 \leq x \leq e \) is

(A) 0.772 (B) 1.221 (C) 1.359 (D) 1.790 (E) 2.097

79. Let \( f(x) = \int_0^x (1 - 2\cos^3 t) \, dt \) for \( 0 \leq x \leq 2\pi \). On which interval is \( f \) increasing?

(A) \( 0 < x < \pi \) (B) \( 0.654 < x < 5.629 \) (C) \( 0.654 < x < 2\pi \)

(D) \( \pi < x < 2\pi \) (E) none of these

80. The table shows the speed of an object (in ft/sec) during a 3-sec period. Estimate its acceleration (in ft/sec^2) at \( t = 1.5 \) sec.

<table>
<thead>
<tr>
<th>time, sec</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed, ft/sec</td>
<td>30</td>
<td>22</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

(A) -17 (B) -13 (C) -10 (D) -5 (E) 17
81. A maple-syrup storage tank 16 ft high hangs on a wall. The back is in the shape of the parabola \( y = x^2 \) and all cross sections parallel to the floor are squares. If syrup is pouring in at the rate of 12 \( \text{ft}^3/\text{hr} \), how fast (in \( \text{ft/hr} \)) is the syrup level rising when it is 9 ft deep?

\[
\text{(A) } \frac{2}{27} \quad \text{(B) } \frac{1}{3} \quad \text{(C) } \frac{4}{3} \quad \text{(D) } 36 \quad \text{(E) } 162
\]

82. In a protected area (no predators, no hunters), the deer population increases at a rate of \( \frac{dP}{dt} = k(1000 - P) \), where \( P(t) \) represents the population of deer at \( t \) yr. If 300 deer were originally placed in the area and a census showed the population had grown to 500 in 5 yr, how many deer will there be after 10 yr?

\[
\text{(A) } 608 \quad \text{(B) } 643 \quad \text{(C) } 700 \quad \text{(D) } 833 \quad \text{(E) } 892
\]

83. Shown is the graph of \( f(x) = \frac{4}{x^2 + 1} \).

Let \( H(x) = \int_0^x f(t) \, dt \). The local linearization of \( H \) at \( x = 1 \) is \( H(x) \) equals

\[
\text{(A) } 2x \quad \text{(B) } -2x - 4 \quad \text{(C) } 2x + \pi - 2
\]
\[
\text{(D) } -2x + \pi + 2 \quad \text{(E) } 2x + \ln 16 + 2
\]
84. A smokestack 100 ft tall is used to treat industrial emissions. The diameters, measured at 25-ft intervals, are shown in the table. Using the midpoint rule, estimate the volume of the smokestack to the nearest 100 ft$^3$.

<table>
<thead>
<tr>
<th>ht</th>
<th>dia</th>
</tr>
</thead>
<tbody>
<tr>
<td>100'</td>
<td>5'</td>
</tr>
<tr>
<td>75'</td>
<td>7'</td>
</tr>
<tr>
<td>50'</td>
<td>9'</td>
</tr>
<tr>
<td>25'</td>
<td>15'</td>
</tr>
<tr>
<td>0'</td>
<td>17'</td>
</tr>
</tbody>
</table>

(A) 8100  (B) 9500  (C) 9800  (D) 12,500  (E) 39,300

For Questions 85–89 the table shows the values of differentiable functions $f$ and $g$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
<th>$f'$</th>
<th>$g$</th>
<th>$g'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>-3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

85. If $P(x) = \frac{f(x)}{g(x)}$, then $P'(3) =$

(A) $-2$  (B) $-\frac{8}{9}$  (C) $-\frac{1}{2}$  (D) $\frac{2}{3}$  (E) 2

86. If $H(x) = f'(g(x))$, then $H'(3) =$

(A) 1  (B) 2  (C) 3  (D) 6  (E) 9

87. If $M(x) = f(x) \cdot g(x)$, then $M'(3) =$

(A) 2  (B) 6  (C) 8  (D) 14  (E) 16

88. If $K(x) = g^{-1}(x)$, then $K'(3) =$

(A) $-\frac{1}{2}$  (B) $-\frac{1}{3}$  (C) $\frac{1}{3}$  (D) $\frac{1}{2}$  (E) 2
89. If \( R(x) = \sqrt{f(x)} \), then \( K'(3) = \)

(A) \( \frac{1}{4} \)  \hspace{1cm}  (B) \( \frac{1}{2\sqrt{2}} \)  \hspace{1cm}  (C) \( \frac{1}{2} \)  \hspace{1cm}  (D) \( \sqrt{2} \)  \hspace{1cm}  (E) 2

90. Water is poured into a spherical tank at a constant rate. If \( W(t) \) is the rate of increase of the depth of the water, then \( W \) is

(A) constant  \hspace{1cm}  (B) linear and increasing  \hspace{1cm}  (C) linear and decreasing
  \hspace{1cm}  (D) concave up  \hspace{1cm}  (E) concave down

91. The graph of \( f' \) is shown below. If \( f(7) = 3 \) then \( f(1) = \)

![Graph of f']

(A) \( -10 \)  \hspace{1cm}  (B) \( -4 \)  \hspace{1cm}  (C) \( -3 \)  \hspace{1cm}  (D) \( 10 \)  \hspace{1cm}  (E) \( 16 \)

92. At an outdoor concert, the crowd stands in front of the stage filling a semicircular disk of radius 100 yd. The approximate density of the crowd \( x \) yd from the stage is given by

\[
D(x) = \frac{20}{2\sqrt{x} + 1}
\]

people per square yard. About how many people are at the concert?

![Semicircle Diagram]

(A) \( 200 \)  \hspace{1cm}  (B) \( 19,500 \)  \hspace{1cm}  (C) \( 21,000 \)  \hspace{1cm}  (D) \( 165,000 \)  \hspace{1cm}  (E) \( 591,000 \)

93. The Centers for Disease Control announced that, although more AIDS cases were reported this year, the rate of increase is slowing down. If we graph the number of AIDS cases as a function of time, the curve is currently

(A) increasing and linear
(B) increasing and concave down
(C) increasing and concave up
(D) decreasing and concave down
(E) decreasing and concave up
The graph below is for Questions 94–96. It shows the velocity, in feet per second, for $0 < t < 8$, of an object moving along a straight line.

![Graph showing velocity vs. time](image)

94. The object’s average speed (in ft/sec) for this 8-sec interval was

(A) 0  (B) $\frac{3}{8}$  (C) 1  (D) $\frac{8}{3}$  (E) 8

95. When did the object return to the position it occupied at $t = 2$?

(A) $t = 4$  (B) $t = 5$  (C) $t = 6$  (D) $t = 8$  (E) never

96. The object’s average acceleration (in ft/sec²) for this 8-sec interval was

(A) $-2$  (B) $-\frac{1}{4}$  (C) 0  (D) $\frac{1}{4}$  (E) 1

97. If a block of ice melts at the rate of $\frac{72}{2t+3}$ cm³/min, how much ice melts during the first 3 min?

(A) 8 cm³  (B) 16 cm³  (C) 21 cm³  (D) 40 cm³  (E) 79 cm³

98. A particle moves counterclockwise on the circle $x^2 + y^2 = 25$ with a constant speed of 2 ft/sec. Its velocity vector, $\mathbf{v}$, when the particle is at (3, 4), equals

(A) $-\frac{1}{5}(8i - 6j)$  (B) $\frac{1}{5}(8i - 6j)$  (C) $-2\sqrt{3}i + 2j$

(D) $2i - 2\sqrt{3}j$  (E) $-2\sqrt{2}(i - j)$

99. Let $\mathbf{R} = a \cos kt\mathbf{i} + a \sin kt\mathbf{j}$ be the (position) vector $x\mathbf{i} + y\mathbf{j}$ from the origin to a moving point $P(x, y)$ at time $t$, where $a$ and $k$ are positive constants. The acceleration vector, $\mathbf{a}$, equals

(A) $-k^2\mathbf{R}$  (B) $a^2k^2\mathbf{R}$  (C) $-a\mathbf{R}$

(D) $-ak^2\cos t\mathbf{i} + \sin t\mathbf{j}$  (E) $-\mathbf{R}$

100. The length of the curve $y = 2^t$ between (0, 1) and (2, 4) is

(A) 3.141  (B) 3.664  (C) 4.823  (D) 5.000  (E) 7.199
101. The position of a moving object is given by \( P(t) = (3t, e^t) \). Its acceleration is
(A) undefined
(B) constant in both magnitude and direction
(C) constant in magnitude only
(D) constant in direction only
(E) constant in neither magnitude nor direction

102. Suppose we plot a particular solution of \( \frac{dy}{dx} \) from initial point (0, 1) using Euler's method. After one step of size \( \Delta x = 0.1 \), how big is the error?
(A) 0.09 (B) 1.09 (C) 1.49 (D) 1.90 (E) 2.65

103. We use the first three terms to estimate \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1} \). Which of the following statements is (are) true?

I. The estimate is 0.7.
II. The estimate is too low.
III. The estimate is off by less than 0.1.

(A) I only (B) III only (C) I and II (D) I and III (E) all three

104. Which of these diverges?

(A) \( \sum_{n=1}^{\infty} \frac{2}{3^n} \) (B) \( \sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^n \) (C) \( \sum_{n=1}^{\infty} \frac{2}{3n} \)
(D) \( \sum_{n=1}^{\infty} \frac{2}{n^3} \) (E) \( \sum_{n=1}^{\infty} \frac{2n}{n^n} \)

105. Find the radius of convergence of \( \sum_{n=1}^{\infty} \frac{n!}{n^n x^n} \).

(A) 0 (B) (C) 1 (D) \( e \) (E) \( n \)

106. When we use \( e^x = 1 + x + \frac{x^2}{2} \) to estimate \( \sqrt{e} \), the Lagrange remainder is no greater than

(A) 0.021 (B) 0.034 (C) 0.042 (D) 0.067 (E) 0.742

107. An object in motion along a curve has position \( P(t) = (\tan t, \cos 2t) \) for \( 0 \leq t \leq 1 \). How far does it travel?

(A) 0.96 (B) 1.73 (C) 2.10 (D) 2.14 (E) 3.98